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Article

Analysis of Solutions of Linear System Using the Gaussian Elimination Method for Production Optimization in the Bakery Industry

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A B S T R A C T

The bakery industry faces challenges in optimizing production due to limited raw materials and multiple product types. This study aims to analyze a system of linear equations representing the relationship between product quantities and raw material usage using the Gaussian elimination method. A case study involving bread, sweet bread, and cake production was conducted based on the availability of flour, sugar, and eggs. The system was formulated and solved using Gaussian elimination to obtain its general solution. The results show that the system has infinitely many solutions, indicating multiple feasible production combinations that fully utilize available resources. One practical solution identified is the production of 20 units of bread, 30 units of sweet bread, and 20 units of cake. This combination ensures optimal use of raw materials without waste. The findings demonstrate that Gaussian elimination is an effective method for supporting production planning and decision-making. Overall, linear algebra provides a reliable approach for optimizing resource allocation in the bakery industry.

Keywords: *Bakery Industry, Gaussian Elimination, Linear Equation System, Production Optimization, Raw Materials*

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INTRODUCTION

The bread and cake industry is part of the food industry sector that requires efficient production planning mechanisms to meet consumer demand and make optimal use of resources (Heizer et al., 2020; Stevenson, 2018). In the production process, a common problem arises is determining the amount of each product that must be produced so that the limited raw materials can be utilized optimally (Render & Heizer, 2018). Inaccurate planning can lead to waste of raw materials, increased operational costs, and an imbalance between the needs and availability of products (Nahmias, 2009; Sule, 2008).

One of the widely used approaches to overcome production optimization problems is mathematical modeling using a linear equation system. In the context of optimization, a linear system is able to represent the relationship between decision variables and resource constraints in a structured and quantitative manner (Hillier & Lieberman, 2021). Previous research has shown that linear models are effective in aiding production planning and resource allocation in various industries, including the food industry (Alemayehu & Zhu, 2019; Mula et al., 2006). In addition, the linear equation system is an important part of linear algebra which is widely used to describe the relationships between variables in various fields such as engineering, economics, and computer science (Lay, 2016; Strang, 2016).

In solving the system of linear equations, various numerical methods have been developed, one of which is the Gauss Elimination method. This method is a basic technique in linear algebra that is used to reduce the matrix to a linear echelon shape so that a solution can be obtained systematically (Anton & Rorres, 2014). The application of the Gauss Elimination method has been widely used in various fields, including scientific computing and production system optimization, due to its efficiency and accuracy in solving linear systems (Chapra & Canale, 2015). Several studies have also shown that linear algebra-based approaches can be used effectively to solve optimization problems in an industrial context (Sharma & Sharma, 2020; Tiwari & Saxena, 2018).

However, most previous research has focused more on the use of more complex optimization methods such as linear programs and advanced optimization techniques, while the use of basic methods such as Gauss Elimination in the context of production decision-making is still relatively limited (Bertsimas & Tsitsiklis, 1997; Dantzig, 1998). In addition, research that specifically examines the interpretation of linear equation system solutions—especially infinite solutions—in relation to production flexibility in the bakery industry is still rare.

Therefore, there is a *research gap* in the application of the Gauss Elimination method as a simple but effective analytical tool to support production decision-making, especially in understanding the practical meaning of linear equation system solutions. This study seeks to fill this gap by analyzing the solution of the linear equation system using the Gauss Elimination method and interpreting the results in the context of production optimization in the bakery industry.

Thus, this research is expected not only to make a theoretical contribution in the field of linear algebra, but also to provide practical implications for production management in determining product combinations that are efficient, flexible, and in accordance with the availability of raw materials (Heizer et al., 2020; Hillier & Lieberman, 2021)

METHOD

Research Approach

This study uses a quantitative approach with a mathematical analysis method based on a linear equation system to model the relationship between the amount of production and the use of raw materials (Sugiyono, 2019). This approach is commonly used in production optimization because it is able to represent variable relationships in a structured and quantitative manner (Hillier & Lieberman, 2021).

Variable Definition

For example:

x = amount of white bread production (units)
y = total sweet bread production (units)
z = total cake production (units)

The definition of these variables is the first step in mathematical modeling to connect decision variables with available resources (Taha, 2017).

Modeling of Linear Equation Systems

Based on the need for raw materials (flour, sugar, and eggs), the following linear equation system is obtained:

$$\begin{cases} 200x + 150y + 100z = 10500 \\ 20x + 50y + 80z = 3500 \\ x + 2y + 3z = 140 \end{cases}$$

This model represents resource limitations in the form of mathematical equations, which is a common approach in production optimization (Alemayehu & Zhu, 2019; Mula et al., 2006). The linear equation system is also the basis for the analysis of the relationship between variables in various fields of engineering and industry (Lay, 2016).

The system is expressed in the form of an augmented matrix :

$$\left[\begin{array}{ccc|c} 200 & 150 & 100 & 10500 \\ 20 & 50 & 80 & 3500 \\ 1 & 2 & 3 & 140 \end{array}\right]$$

Matrix representation is used to facilitate algebraic manipulation and computational processes (Strang, 2016).

Linear Equation System Solutions

The first step is to form an augmented matrix of a system of linear equations. The step of forming an augmented matrix from a system of linear equations is a basic procedure in linear algebra to simplify the solution of the system (Lay, 2016).

Gauss eliminated

The steps of applying the Gauss Elimination method are as follows: (a) the application of the Gauss Elimination method is carried out by determining pivots, performing elementary line operations, and changing the matrix to the form of the upper line echelons (Anton and Rorres, 2014); (b) define the pivot (main element) on the first line; (c) perform elementary line operations to eliminate variables under pivot; (d) change the matrix to the top-row echelon form; (e) perform reverse substitution to obtain a solution, where this process is used to obtain a solution from the system after the matrix is in the echelon form (Chapra and Canale, 2015); and (f) determine realistic (positive and production-compliant) special solutions (Taha, 2017).

Solution Analysis

Solution analysis is carried out to determine whether the system has a single, infinite, or inconsistent solution (Lay, 2016), with the following steps: (a) determining whether the system has a single, infinite, or inconsistent solution; (b) identify the optimal production combination of the solution line if the system has infinite solutions; and (c) make practical interpretations of the solution results to provide production recommendations, where these interpretations are used as a basis in production decision-making (Heizer et al., 2020).

Research Flow (Diagram)

Collection of raw material data and product recipes → Formulation of a system of linear equations → Formation of augmented matrix → Completion using the Gauss Elimination method → Analysis of solutions and interpretation → Recommendation of optimal production combinations

Data Validation and Analytics

The verification process is carried out through several stages as follows: (a) data is checked for consistency and mathematical conformity with the recipe and total raw materials; (b) the solution is tested by substitution back into the equation system to ensure consistency of raw material use; and (c) the resulting solutions are verified to ensure that the production quantity is positive and realistic.

FINDINGS AND DISCUSSION

Findings

Early Matrix Models

Equation system: $$\begin{cases} 200x + 150y + 100z = 10500 \\ 20x + 50y + 80z = 3500 \\ x + 2y + 3z = 140 \end{cases}$$

Augmented matrix shape: $$\left[\begin{array}{ccc|c} 200 & 150 & 100 & 10500 \\ 20 & 50 & 80 & 3500 \\ 1 & 2 & 3 & 140 \end{array}\right]$$

Gauss Elimination Steps

Step 1: Switch Rows (so that pivot = 1)

Convert R1 to R3

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 140 \\ 20 & 50 & 80 & 3500 \\ 200 & 150 & 100 & 10500 \end{array}\right]$$

Step 2: Elimination of the First Column

R2 = R2 – 20R1

R2 = (20, 50, 80, 3500) – 20(1,2,3,140)

= (0.10, 20, 700)

R3 = R3 – 200R1

R3 = (200, 150, 100, 10500) – 200(1,2,3,140)

= (0, -250, -500, -17500)

Results : $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 10 & 20 \\ 0 & -250 & -500 \end{bmatrix}\left|\begin{matrix} 140 \\ 700 \\ -17500 \end{matrix}\right]$

Step 3 : Simplify the Lines

R2 : 10

$$(0,1,2,70)$$

R3 : -250

$$(0,1,2,70)$$

Results :

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\left|\begin{matrix} 140 \\ 70 \\ 70 \end{matrix}\right]$$

Step 4 : Elimination of the 3rd row

R3 = R3 – R2

$$(0,0,0,0)$$

Final result (echelon form) :

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\left|\begin{matrix} 140 \\ 70 \\ 0 \end{matrix}\right]$$

Substitusi Balik

From the 2nd line:

$$y + 2z = 70$$
$$y = 70 - 2z$$

From the 1st line:

$$x + 2y + 3y = 140$$

Substitution :

$$x + 2(70 - 2z) + 3z = 140$$
$$x + 140 - 4z + 3z = 140$$
$$x - z = 0$$
$$x = z$$

Common Solutions

$$x = z$$
$$y = 70 - 2z$$

Defining a Custom (Realistic) Solution

To make the production reasonable (not negative, x 0), take:>

$$z = 20$$

So:

$$x = 20$$
$$y = 30$$
$$z = 20$$

Final Results

Optimal production quantity:

Roti tawar = 20 unit
Roti manis = 30 unit
Cake = 20 units

Discussion

Dependent Equations

Solution Infinite solutions occur because the system of linear equations that are formed is not linear dependence. This can be seen from the result of Gauss elimination which produces a zero line (0 = 0) in the line echelon matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\left|\begin{matrix} 140 \\ 70 \\ 0 \end{matrix}\right]$$

The third line in which all elements are zero indicates that one of the equations in the system is a linear combination of other equations, so it does not provide new additional information (Lay, 2016; Strang, 2016). Mathematically, this causes the sum of independent equations to be 2, while the sum of the variables is 3. Since the number of variables is more than the number of independent equations, the system has one independent variable, which produces a parametric solution rather than a single solution (Anton & Rorres, 2014). Thus, the solution does not appear because the system is in an under-determined system, which geometrically represents a line in a three-dimensional space, not a single solution point (Strang, 2016).

Based on the results of the completion of the linear equation system using the Gauss Elimination method, it was obtained that the system formed was consistent (Anton & Rorres, 2014). This is shown by the appearance of a zero line in the elimination process, which means that there are no contradictions in the system (Lay, 2016) and all equations can be fulfilled simultaneously. However, the system does not have a single solution but has an infinite number of solutions, because the number of variables is more than the number of free equations (Lay, 2016) generated after elimination. Thus, the system falls under the category of indeterminate consistent systems (Anton & Rorres, 2014).

Mathematically, this condition indicates that the solution of the system forms a linear relationship that can be expressed in parametric form (Strang, 2016). One of the variables becomes an independent variable, while the other variable is expressed as a function of the variable. This indicates that the solution of the system is not a single point, but a line in a three-dimensional space. Each point on the line represents a combination of variable values that meet all equations in the system (Anton & Rorres, 2014).

In the context of the bakery industry, this result has a very important meaning, the existence of flexibility in determining the production combination (Heizer et al., 2020). The existence of a solution does not show that there are many combinations of white bread, sweet bread, and cakes that can make optimal use of raw materials without residue (Taha, 2017). In other words, producers have the flexibility to determine the amount of production according to market needs, consumer preferences, or business strategies applied (Stevenson, 2018). This provides an advantage because production does not have to be fixated on one specific combination, but can be adjusted to real conditions in the field.

From the various possible solutions available, one of the special solutions was chosen, namely a combination of production (20, 30, 20) as the most realistic solution. This selection is based on several considerations, namely all variable values have positive values, are easy to implement on a production scale, and have a balanced proportion between products (Taha, 2017). In addition, this combination has also been verified by substituting back into the equation system and proven to be able to use all raw materials optimally without excess or deficiency (Hilier & Lieberman, 2021). Therefore, the solution is considered the most suitable representation to be applied in production planning in the bakery industry.

Interpretation of economic meaning

In the context of the bakery industry's production economy, the existence of unlimited solutions has an important meaning in terms of the flexibility of resource allocation. This system shows that not only can one production combination consume all raw materials, but there are various alternative production combinations that are all resource-efficient (Hillier & Lieberman, 2021). Economically, this means:

Production Flexibility

The company is not tied to one fixed production pattern. Manufacturers can adjust the quantity of white bread, sweet bread, and cakes according to market demand, consumer preferences, sales strategy. This increases responsiveness to the market (Heizer et al., 2020).

Resource Usage Efficiency

Each solution in the system ensures that all raw materials (flour, sugar, eggs) are used to the fullest with no residues or shortages. This reflects the optimal resource efficiency condition, which is the condition when inputs are used optimally without waste (Taha, 2017).

Trade-offs Between Products

Because the solution is parametric:

$$x = t$$
$$y = 70 - 2t$$
$$z = t$$

then there is a substitution relationship between products. This means that increasing the production of one type of product will reduce other products and there is a production trade-off between product lines. This is common in operations management, where limited resources must be optimally allocated between multiple products (Render & Heizer, 2018).

Managerial Decision Making Policy

The solution does not leave room for production managers to choose the most suitable combination for additional business objectives such as: maximum profit, highest market demand, or distribution capacity. Thus, mathematical solutions are not only theoretical, but also the basis for flexible decision-making in production planning (Hillier & Lieberman, 2021).

CONCLUSIONS

Overall, this study shows that modeling the linear equation system using the Gauss Elimination method is able to provide a clear picture of the relationship between production variables in the bakery industry. The results of the analysis indicate that the system built is consistent but does not have a single solution, but rather produces a parametric solution that reflects the existence of various alternative production combinations that are equally efficient in the use of resources. This condition provides an important implication that the production process can be flexibly regulated without sacrificing raw material optimization. Of the many possibilities available, the selection of a single production combination can still be done based on practical considerations such as output balance, ease of implementation, and suitability to operational needs. Thus, this mathematical approach is not only useful for obtaining technical solutions, but also supports more adaptive managerial decision-making in production planning.

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